

Black Hole Entropy: Linear Or Quadratic?

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The entropy S of any mass M can be interpreted as a linear relation with mass, $S \simeq kM/m_g$ an extensive property with m_g the mass of the quantum of gravity. One can extend this relation to black holes and then we get a different result, as compared with the “standard” relation $S \simeq k/\hbar c GM^2$ which is quadratic in the mass. We discuss both approaches and apply it to cosmology.

1 Introduction

We will refer to three concepts already present in the scientific literature: the gravity quanta [1], the gravitational cross section and the gravitational entropy [2]. The first is a step in the direction of introducing quantum mechanical concepts into general relativity and cosmological treatments. From general relativity we know that the gravitational field (energy) must be unlocalized, and this must be the case for the gravity quanta. We impose then the geometrical condition that the Compton wavelength of the gravity quanta of mass m_g must be of the order of the size of the seeable Universe ct , where t is the age of the Universe:

$$\frac{\hbar}{m_g c} \simeq ct \quad \implies \quad m_g \simeq \frac{\hbar}{c^2 t} \simeq 2 \cdot 10^{-66} \text{ gr} \quad (1)$$

The second concept, the gravitational cross section, is derived from Newton’s gravitational force expressed as an interchange of momenta between gravitational masses: M gravitating on m . The mass M has acquired through the age of the Universe t a “well” of momentum Mc . This can act on an effective gravitational cross section σ_g of the mass m in accordance with the geometrical proportion $\sigma_g/4\pi r^2$. This mechanical definition gives

$$\frac{GMm}{r^2} \simeq \frac{Mc}{t} \cdot \frac{\sigma_g}{4\pi r^2} \quad (2)$$

and then

$$\sigma_g \simeq 4\pi \frac{Gm}{c^2} \cdot ct \quad (3)$$

We see that the gravitational cross section of a mass m is of the order of the product of its gravitational radius times the size of the seeable Universe. As a reference, particularizing for a proton, its gravitational cross section is of the order of its area,

as defined by the square of its Compton wavelength. On the other hand, the gravitational cross section for the whole Universe of mass M_u is the square of its size $ct \simeq GM_u/c^2$ as given by Mach's principle in the way we will interpret later. Then one has [2]

$$\sigma_g \simeq 4\pi \frac{Gm}{c^2} \cdot \frac{GM_u}{c^2} \quad (4)$$

The conclusion is that the gravitational cross section of any mass m is of the order of the product of two gravitational radii as given by its mass and the mass of the Universe.

Following an approach similar to Hawking [3] and Bekenstein [4], and using our gravitational cross section concept (4) we define entropy S of any mass m as

$$\frac{S}{k} \simeq \frac{\sigma_g}{A} \quad (5)$$

where A is the Plancks area. Using (4) we get then

$$\frac{S}{k} \simeq 4\pi \frac{Gm}{c^2} \cdot ct \cdot \frac{c^3}{G\hbar} = 4\pi \frac{mc^2t}{\hbar} = 4\pi \frac{m}{m_g} \quad (6)$$

The dimensionless entropy S/k of a mass m is the number of gravity quanta emitted: this definition of entropy is an extensive property and is linear with mass. This is a gravitational entropy.

2 Standard Black Hole Entropy Versus the New Approach

The Hawking [3] and Bekenstein [4] black hole entropy is given by

$$\frac{S}{k} \simeq \left(\frac{Gm}{c^2} \right)^2 \cdot \frac{c^3}{G\hbar} = \frac{1}{\hbar c} Gm^2 \quad (7)$$

which is quadratic in the mass and it has no time dependence. If we apply it to the mass of the Universe, considering it as a kind of black hole, we get

$$\frac{S}{k} \simeq \frac{1}{\hbar c} GM_u^2 \quad (8)$$

Our entropy definition (6) implies proportionality with mass and cosmological time. Our black hole entropy is from (6)

$$\frac{S}{k} \simeq 4\pi \frac{mc^2t}{\hbar} \quad (9)$$

and for the Universe

$$\frac{S}{k} \simeq 4\pi \frac{M_u c^2 t}{\hbar} \quad (10)$$

again linear with mass and cosmological time. We will transform this relation taking into account a definition of Machs principle to include cosmology.

3 Machs Principle

We interpret Machs Principle by saying that the gravitational potential energy of any mass m , with respect to the rest of the Universe, is its rest energy (which can be related to one form of the Equivalence Principle). Hence

$$\frac{GM_u m}{ct} \simeq mc^2 \quad (11)$$

i.e.

$$\frac{GM_u}{c^2} \simeq ct \quad (12)$$

Then using this we get our black hole gravitational entropy from (10)

$$\frac{S}{k} \simeq \frac{4\pi}{\hbar c} GM_u^2 \quad (13)$$

which is the same as the Hawking-Bekenstein relation (8) for the Universe.

4 Conclusions

The gravitational entropy of a mass m can be defined as the number of gravity quanta emitted. It is proportional to mass and cosmological time and therefore it is an extensive property, something desired for the concept of entropy.

This entropy can be applied to BLACK HOLES, and the result is in contrast with the Hawking-Bekenstein relation (that is quadratic in mass).

For the cosmological case, for the Universe using Mach's Principle, both approaches give the same result.

The gravitational entropy of the Universe, as we have defined, is then

$$\frac{S}{k} \simeq \frac{GM_u^2}{\hbar c} \simeq 10^{122} \quad (14)$$

and we conclude that this number is so huge because the Universe is very old.

References

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- [4] J. Bekenstein, *J. Phys. Rev.*, 1972, D7, 2333-2346.